RAN-2203000206020036

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T. Y. B. Sc. (Sem. - VI) Examination September - 2025

Mathematics : MTH-606 (Paper - XI) Number Theory - II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **T. Y. B. Sc. (Sem. - VI)**

Name of the Subject :

☛ **Mathematics : MTH-606 (Paper - XI) Number Theory - II**

Subject Code No.: **2203000206020036**

Seat No.:

Student's Signature

Q. 1. Answer any Five of the following: (10)

1. State the Chinese Reminder Theorem.
2. Converse of Fermat's little theorem is true? Justify your answer?
3. Verify $\sigma(n)=\sigma(n+1)$ for $n=206$.
4. Find the value of $\phi(780)$.
5. Find a solution of the linear congruence $36x \equiv 8 \pmod{102}$.
6. Find the reminder when 15! is divided by 17.
7. Prove that $[x]+[-x]=-1$ where x is not an integer.
8. Prove that $\phi(n)=\phi(2n)$ for $n=p(2p-1)$ where p and $2p-1$ both are odd primes.

Q. 2. Answer any Two questions: (10)

1. Solve the linear congruence $25x \equiv 15 \pmod{29}$.
2. Find an integer having reminder 1, 2, 3 when divided by 3, 5, 7 respectively.
3. Find an integer a such that $2^2 \mid a$, $3^2 \mid a+1$, $5^2 \mid a+2$.

Q. 3. **Answer any Two questions:** **(10)**

1. If p and q are distinct primes such that $a^q \equiv a(\text{mod}\, p)$ and $a^p \equiv a(\text{mod}\ q)$ then $a^{pq} \equiv a(\text{mod}\ pq)$.
2. Verify that $4(29!)+5!$ is divisible by 31.
3. Find the unit digit of 3^{100} using Fermat's little theorem.

Q. 4. **Answer any Two questions:** **(10)**

1. If 2^k-3 is prime then show that $n = 2^{k-1}(2^k-3)$ satisfies $\sigma(n) = 2n+2$.
2. For any integer $n \geq 3$, prove that $\sum_{k=1}^{n} \mu(k!) = 1$
3. Find the number of zeros with which 1000! terminates.

Q. 5. **Answer any Two questions:** **(10)**

1. Prove that $\phi(3n) = 3\phi(n)$ if and only if $3 \mid n$.
2. If n is an odd integer such that $5 \nmid n$ then show that n divides the integer whose all digits are 1.
3. For $n \geq 2$, prove that $\phi(n)$ is an even integer.
